Two World Oil Production Parabolas Staged at 1988

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Two Staged Parabolas based upon ultimate resources of conventional oil of 2200 and 3000 gigabarrels (GB) respectively were plotted for world production. In both cases the parabolas were staged at 1988 and their shapes defined by the production in 1997. The calculation thus depended only upon the cumulative production to the end of 1988 and the known production in 1988 and 1997. For a resource base of 2200 GB, the peak was found to be 28.15 GB in mid-2010 and for the 3000 GB case, 30.29 GB in mid-2022. No other information was needed to draw the two parabolas. Because their shape does not depend upon the historical patterns of production over time, the parabolas reflect the future and not the past. Since the 3000 GB case was included as an extreme upper bound, the 2200 GB case was considered the more likely unless there are major unexpected discoveries. Extended Parabolas were also drawn for the two cases to illustrate the effect of increased recovery from known in-situ resources after the peak in production from conventional sources is past when prices are expected to increase significantly

Introduction

The course of future oil production from conventional sources is uncertain yet there should be a general underlying pattern in the sense some production-time relationship exhibiting a peak no doubt exists. The issue is how best future oil production on a world basis can be forecast.

The attributes of oil suggest production may be modelled mathematically to a reasonable approximation of reality. Oil or its products can fill almost all energy needs and, as transportation costs by tanker on the sea or by pipeline on land are low, its price may be not greatly different anywhere on the main trading routes. The consequence is that oil may fill as much of the world's energy needs as its owners may desire by lowering its price. Given these characteristics, the physical consumption of oil should fluctuate less with economic conditions than the other fossil fuels which oil may displace. It follows that global oil production should be more stable and reflect the underlying resource base more closely than other fossil fuels.

Given that oil production will tend to reflect resource constraints, mathematical techniques may be used to project future oil production based upon the inherent characteristics of large numbers. A strong case can be made that a parabolic curve may best forecast the future pattern of oil production particularly around the peak period. For the same reason, the behaviour of the oil supply system and, more generally, the fossil fuel energy supply system, will change radically once the peak of low-cost oil production from conventional sources has passed – a particularly important point should the emissions of carbon dioxide and other greenhouse gases be controlled from the fossil fuels.

Based upon two geological assessments of the ultimate recovery of conventional oil of 2200 and 3000 gigabarrels (GB) respectively, the cumulative production of world oil to 1988 is used to draw two Staged Parabolas. The basis of the Staged Parabola is explored in detail in a companion Technical Note[1]. Unlike other parabolic projection techniques, the shape of the Staged Parabola does not depend upon the past time-series pattern of production; only the cumulative and annual production is employed for the year chosen as the basis of calculation (1988) though some additional piece of information about the future after the staging date is required to completely define its shape. It is in this sense, a forward-looking parabola. Here, the Staged Parabola is set to pass through the actual production for 1997 for which year cumulative output is also known. Hence, only the cumulative and yearly production in those two years are involved in drawing the parabolas for the two estimated cases of the ultimate recoverable endowment of world conventional oil resources. These two cases were chosen because, according to authorities cited below, 2200 GB represents a median estimate of the world endowment and 3000 GB an outer bound. The actual endowment is probably closer to 2200 GB but the second case covers the situation in the eventuality that much more oil is discovered than expected with the advent of the new discovery and production technologies. The projected tracks of the two parabolas drawn here are thus independent of any other information.

These results may be compared with the Staged Parabolas for the world production of oil that appear in the earlier Technical Note. In the previous case, the staging year chosen was 1996 and the future reference point was taken from a scenario formulated by the International Energy Agency in which the peak production of the world was estimated to be 28.8 GB. The present case uses an earlier date for the staging – 1988 – because a

[1] J.H. Walsh, Technical Note: *Comparing Parabolic and Normal Curves for Modelling the Peak Period in Petroleum Production*, July 1998. Copies are available from the author

reliable value of the cumulative production to the end of that year was available of 610.1 GB[2]. Cumulative production is also known to the end of 1997 by adding the production in that year to the value of 807 GB to the end of 1996 given by Campbell and Leherrère[3].

Methodology

The methodology employed is presented in detail in this note so that the calculations are as transparent as possible. Those holding different views of oil production may employ the same parabolic techniques in accordance with their own assessments of the resource base.

Consistency of Data Used

Since cumulative and yearly production data are available for both the years chosen as the basis of these calculations – staging at 1988 with passage of the resulting parabola through the 1997 production point-self-consistency is important. This problem arises because oil production in any given year may vary for a number of extraneous factors such as commercial, political or accidental short-term conditions such as bad weather in off-shore producing regions. The simplest check is to represent the cumulative production for the nine years separating the two selected dates on the production-time parabola as the sum of the area of a rectangle and a triangle drawn between these years as in the following equation:

$$(P_{1998} \times 9) + \{½ (p_{1997} - p_{1998})\} \times 9$$
$$= (23.09 \times 9) + ½ \times 9(26.36 - 23.09)$$
$$= 222.525 \text{ GB}$$

The sum of the two rectilinear geometric figures of 222.53 GB may be compared with the actual change in cumulative production between these nine years, which is 223.26 GB. This estimate is close at 0.33% less than the actual value, but the agreement would be even greater given that the concave shape of the parabolic curve extends outwards between the two production points which increases the area not captured by the long side of the triangle. For this reason, this check is considered effective and the basic data used to derive the parabola is thought reasonably self-consistent.

Reiterative Calculation Technique

The solution followed the technique developed in the Technical Note[4]. The area under the Staged Parabola was designated Q_s and its proportion in the overlap section was expressed as a different percentage of Q_s for each iteration. It is thus possible to derive values for Q_s and $q_{overlap}$ for the assumed percentage for each trial. The values for the final iteration in the 2200 GB case with $q_{overlap}$ assumed to be the correct value of 20.1% of Q_s follow as an illustration:

$q_{1988} + Q_s - q_{overlap} = Q_u$, the ultimate resource potential estimated at 2200 GB. Hence:

$$Q_s = (2200 - 610.1)/(1 - 0.201) = 1989.86 \text{ GB}$$

and:

$$q_{overlap} = Q_s \times 0.201 = 399.96 \text{ GB}$$

With a knowledge of $q_{overlap}$ and Q_s it is possible to derive r_1, the ratio of the time t_1 from the zero of start of the parabola to T, its total time given by the two intersections of the parabola with the time axis, by solving the following cumulative production equation:

$$q_{overlap} = 3Q_s r_1^2 - 2Q_s r_1^3$$

This equation may be solved by the use of a mathematics program or by hand methods involving no more that about 12 iterations to reach sufficient accuracy. Since the value of r is defined to lie between 0 and 1, for the case of mathematics programs giving only three decimal figures after the zero, it is convenient to substitute $x_1 = 100r_1$, which will produce a result with five significant figures in most cases.

With knowledge of r_1, it is possible to derive the value of T from the re-arranged production-time equation:

$$T = (6\, Q_s/p_1)\, r_1\, (1 - r_1) = 106.02 \text{ years}$$

where p_1 is the production in 1988 of 23.09 GB.

With knowledge of T, it is possible to derive t_1 as 30.53 years, and, by adding the nine years to 1997, t_2, together with the starting (1957.47) and final calendar years (2063.49) of the Staged Parabola. It is thus possible to calculate the production in the year t_2 from the following production-time equation, which should equal the actual value for this year at the correct iteration

$$p_2 = 6(Q_s/T^2)\, t_2\{1 - (t_2/T)\}$$

Since the cumulative production in 1988 and 1997 is known, this estimate may be checked by using t_2 in the cumulative equation to determine q_{1997} as follows:

$$Q_{1997} = 3\, Q_s r_2^2 - 2\, Q_s r_2^3 = 623.584 \text{ GB}$$

Subtracting the overlap volume of 399.96 GB gives the cumulative production between 1988 and 1997 as 223.62 GB which is to be compared with the actual value of 223.26 GB, a difference of only 0.16% which confirms the solution.

This exercise was repeated in the same way for the 3000 GB case.

[2]. C.D. Masters, D.H. Root and E.D. Attansi, *Resource Constraints in Petroleum Production Potential*, Science, Vol **253**, 12 July 1991

[3] Colin J. Campbell and Jean H. Laherrère, *The End of Cheap Oil*, Scientific American, Vol. **28**, No.3, (March) 1998

[4] loc cit

Staged World Oil Production Parabolas

Figure 1: Underlying Resource Base of 2200 GB

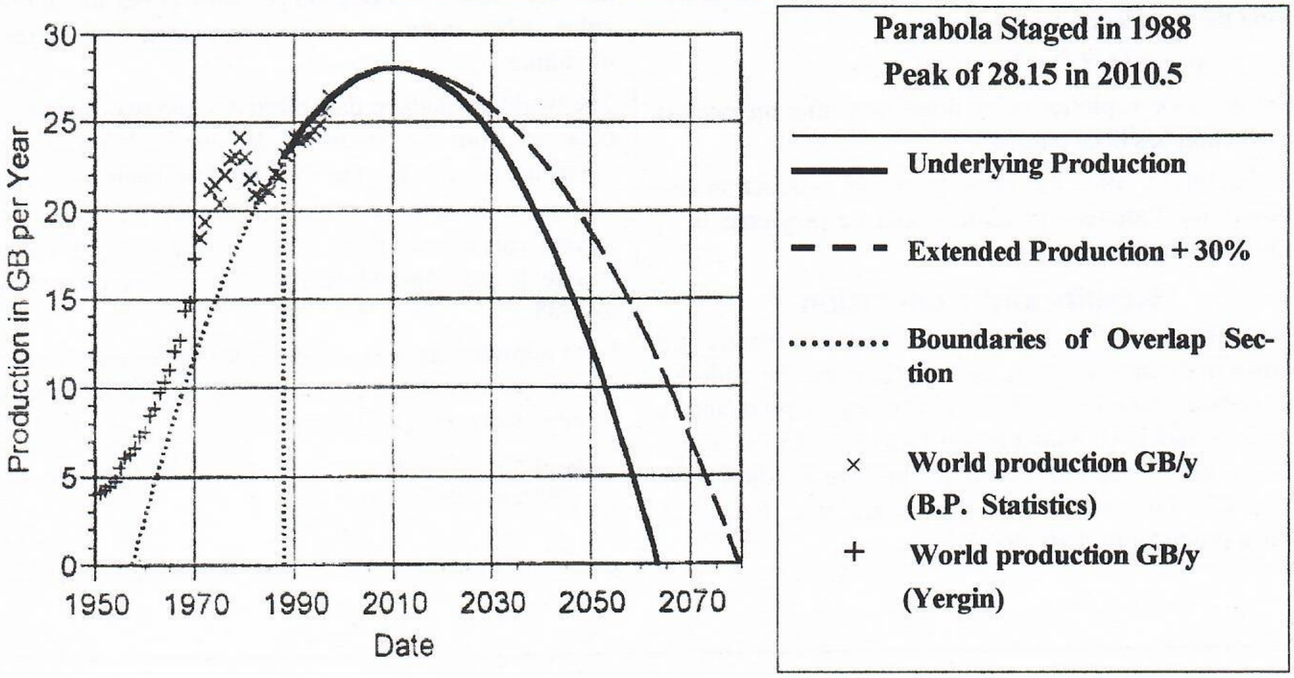
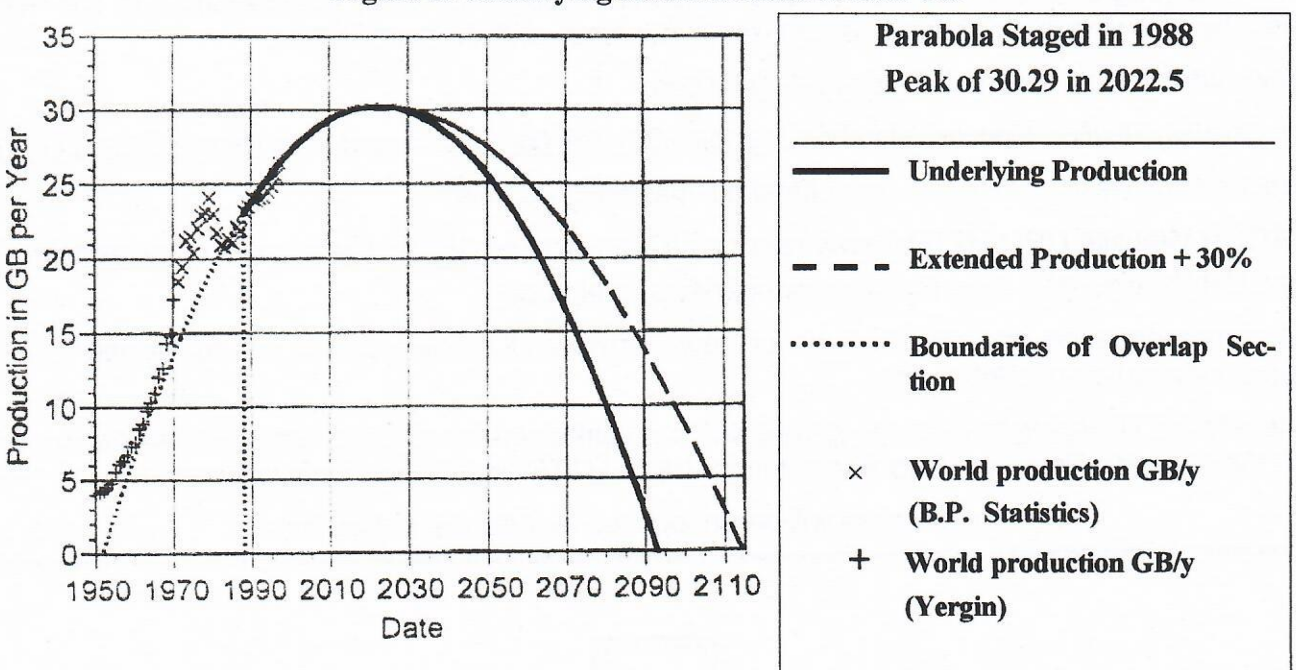


Figure 2: Underlying Resource Base of 3000 GB



Extended Parabola.

An Extended Parabola was included for both cases for reasons explained in more detail in the Technical Note (*loc. cit.*) In essence, after the peak in world oil production is passed, the higher prices to be expected at that time would likely lead to the deployment of the more costly enhanced recovery processes which should result in a higher recovery of the known oil-in-place. In this paper, this additional recovery is assumed to be effectively a 30% increase in the remaining resource base. The Extended Parabola shares the same peak as the underlying parabola, and the new total time, T_e may be easily calculated from the rearrangement of the area equation for the parabola:

$T_e = 1.5Q_e/P$ where Q_e, the extended resource, is $1.3 \times Q_s$ and the peak P is the same as before. With knowledge of T_e and Q_e, the Extended Parabola may then be drawn using the production-time equation:

$$p_t = (6Q_e/T_e^2)\, t\{1 - (t/T_e)\}$$

This equation is plotted using times only after the peak in production has been passed.

In the future, after the peak in world production has passed, the Extended Parabola could be projected from actual data at that time.

Results and Conclusion

The two parabolas with their extended versions are shown in Figures 1 and 2. In Case 1, when the underlying resource base Q_u = 2200 GB, the peak is predicted to occur in mid-2010 with a production of 28.15 GB (77.1 million barrels/day) for that year. In Case 2, when Q_u = 3000 GB, the peak is predicted to occur in mid-2022 with a production of 30.29 GB (83.0 million barrels/day). The actual behaviour of the world conventional oil supply system is expected to lie between these two cases and probably closer to the lower value unless there are major unexpected discoveries in the future.

The world production data plotted in the two figures was obtained from the *Statistical Review of World Energy* published yearly by the British Petroleum p.l.c. and, before 1971, from *The Prize* by Daniel Yergin[5]. In the staging calculation, there is no requirement for the parabolas to pass through the production-time points prior to 1988.

Even allowing for some unexpected large new discoveries, the peak in world oil production should be expected no later than the 2015-2020 period.

[5] Daniel Yergin, *The Prize*, Simon and Schuster, New York, 1991, (ISBN 0-671-50248-4) p. 786.

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